[Journal Publication of International Research for Engineering and Management](#) (JOIREM)
Volume: 04 Issue: 3 | Mar-2026
ISSN (O) 3107-6696

Parameterized Integer Solutions of Quaternary Quadratic Diophantine Equation

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Abstract - This study explores a generalized quadratic Diophantine equation $s^2 + t^2 + r^2 = v^2 - (\mathcal{K} + \mathcal{P})$ in four variables, building on classical approaches to highlight new connections with Krishnamurthy numbers $\mathcal{K}$ and Leyland primes $\mathcal{P}$, considered here up to five-digit values. Different methods of working with the equation are outlined, and integer solutions are obtained through MATLAB computations.

***Key Words*:** Diophantine equation, Quaternary quadratic Diophantine equation, Krishnamurthy numbers, Leyland primes, Parametric solutions.

***MSC 2020*:** 11D09

1.INTRODUCTION

Diophantine equations seek integer solutions to polynomial equations, encompassing linear forms to intricate nonlinear systems that underpin classic and contemporary mathematical challenges [1,2,8,9,11]. Quadratic Diophantine equations in multiple variables hold particular attention, as solving them uncovers links between algebra and number patterns, often yielding practical ways to generate solutions [3-7,10,12-15]. Building upon prior investigations into quaternary quadratic Diophantine equations where solutions vary widely based on parameters.

This study extends that foundation by tackling a generalized quadratic Diophantine equation $s^2 + t^2 + r^2 = v^2 - (\mathcal{K} + \mathcal{P})$ in four variables with Krishnamurthy numbers $\mathcal{K}$ and Leyland primes $\mathcal{P}$ (bounded at five digits), with MATLAB systematically enumerating all integer solutions.

2. Results and Discussion

This section presents the integer solutions obtained for
$s^2 + t^2 + r^2 = v^2 - (\mathcal{K} + \mathcal{P})$ (1)
under three distinct patterns.
Pattern I:
Substituting $v = s + 1$ in (1) yields

$$s = \frac{t^2 + r^2 - 1 + (\mathcal{K} + \mathcal{P})}{2}$$

Thus, the parametric solutions derived above define the full family of integer solutions to the Diophantine equation $s^2 + t^2 + r^2 = v^2 - (\mathcal{K} + \mathcal{P})$. Notably, for each corresponding pair of values $\mathcal{K} \in \{145, 40585\}$ and $\mathcal{P} \in \{17, 593, 32993\}$ the value of s is uniquely determined alongside the associated value t, r, v.

The integer solutions for six equations were systematically computed with t, r ranging between 1 to 5 using MATLAB and presented below.

Equation 1:

t	r	s	v
1	2	83	84
1	4	89	90
2	1	83	84
2	3	87	88
2	5	95	96
3	2	87	88
3	4	93	94
4	1	89	90
4	3	93	94
4	5	101	102
5	2	95	96
5	4	101	102

Fig. 1

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Equation 2:

t	r	s	v
1	2	371	372
1	4	377	378
2	1	371	372
2	3	375	376
2	5	383	384
3	2	375	376
3	4	381	382
4	1	377	378
4	3	381	382
4	5	389	390
5	2	383	384
5	4	389	390

Fig. 2

Equation 3:

t	r	s	v
1	2	16571	16572
1	4	16577	16578
2	1	16571	16572
2	3	16575	16576
2	5	16583	16584
3	2	16575	16576
3	4	16581	16582
4	1	16577	16578
4	3	16581	16582
4	5	16589	16590
5	2	16583	16584
5	4	16589	16590

Fig. 3

Equation 4:

t	r	s	v
1	2	20303	20304
1	4	20309	20310
2	1	20303	20304
2	3	20307	20308
2	5	20315	20316
3	2	20307	20308
3	4	20313	20314
4	1	20309	20310
4	3	20313	20314
4	5	20321	20322
5	2	20315	20316
5	4	20321	20322

Fig. 4

Equation 5:

t	r	s	v
1	2	20591	20592
1	4	20597	20598
2	1	20591	20592
2	3	20595	20596
2	5	20603	20604
3	2	20595	20596
3	4	20601	20602
4	1	20597	20598
4	3	20601	20602
4	5	20609	20610
5	2	20603	20604
5	4	20609	20610

Fig. 5

Equation 6:

t	r	s	v
1	2	36791	36792
1	4	36797	36798
2	1	36791	36792
2	3	36795	36796
2	5	36803	36804
3	2	36795	36796
3	4	36801	36802
4	1	36797	36798
4	3	36801	36802
4	5	36809	36810
5	2	36803	36804
5	4	36809	36810

Fig. 6

Pattern II:

Postulate $v = s + t, \quad r = 2a$ for any integer $a \geq 1$.

Substituting into (1) follows,

$$4a^2 = 2st - (\mathcal{K} + \mathcal{P})$$

This can be rearranged as,

$$2st = 4a^2 + (\mathcal{K} + \mathcal{P})$$

For each choice of parameters $\mathcal{K} \in \{145, 40585\}$ and $\mathcal{P} \in \{17, 593, 32993\}$. Given any a the values of s, t are obtained and the corresponding value of r, v are determined using MATLAB.

Journal Publication of International Research for Engineering and Management (JOIREM)

Volume: 04 Issue: 3 | Mar-2026

ISSN (O) 3107-6696

Equation 1:

a = 1

s	t	r	v
-83	-1	2	-84
1	83	2	84

a = 2

s	t	r	v
-89	-1	4	-90
1	89	4	90

a = 3

s	t	r	v
-99	-1	6	-100
-33	-3	6	-36
-11	-9	6	-20
1	99	6	100
3	33	6	36
9	11	6	20

a = 4

s	t	r	v
-113	-1	8	-114
1	113	8	114

a = 5

s	t	r	v
-131	-1	10	-132
1	131	10	132

Fig.7

Equation 2:

a = 1

s	t	r	v
-371	-1	2	-372
-53	-7	2	-60
1	371	2	372
7	53	2	60

a = 2

s	t	r	v
-377	-1	4	-378
-29	-13	4	-42
1	377	4	378
13	29	4	42

a = 3

s	t	r	v
-387	-1	6	-388
-129	-3	6	-132
-43	-9	6	-52
1	387	6	388
3	129	6	132
9	43	6	52

a = 4

s	t	r	v
-401	-1	8	-402
1	401	8	402

a = 5

s	t	r	v
-419	-1	10	-420
1	419	10	420

Fig.8

Equation 3:

a = 1

s	t	r	v
-227	-73	2	-300
73	227	2	300

a = 2

s	t	r	v
-137	-121	4	-258
121	137	4	258

a = 3

s	t	r	v
-873	-19	6	-892
-291	-57	6	-348
-171	-97	6	-268
19	873	6	892
57	291	6	348
97	171	6	268

Fig.9

Equation 4:

a = 1

s	t	r	v
-257	-79	2	-336
79	257	2	336

a = 2

s	t	r	v
-883	-23	4	-906
23	883	4	906

Fig.10

Equation 5:

a = 1

s	t	r	v
-349	-59	2	-408
59	349	2	408

a = 2

s	t	r	v
-479	-43	4	-522
43	479	4	522

a = 6

s	t	r	v
-291	-71	12	-362
-213	-97	12	-310
71	291	12	362
97	213	12	310

Fig.11

Equation 6:

a = 5

s	t	r	v
-197	-187	10	-384
187	197	10	384

Fig.12

The above figures illustrate few integer solutions of six distinct quaternary quadratic equations derived using MATLAB, with the parameter $a \geq 1$.

Pattern III:

presume $r = v - 1$, in (1) which yields

Journal Publication of International Research for Engineering and Management (JOIREM)
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$$\mathcal{v} = \frac{\mathcal{s}^2+\mathcal{t}^2+1+(\mathcal{K}+\mathcal{P})}{2}$$

For suitable choice of $\mathcal{s}, \mathcal{t}$ the value of $\mathcal{v}$ is found and the corresponding value of $\mathcal{r}$ is determined. Thus, the parametric solution for equation (1) is found for every pair $\mathcal{K} \in \{145, 40585\}$ and $\mathcal{P} \in \{17, 593, 32993\}$ using MATLAB.

The integer solutions of six distinct quaternary quadratic equations were systematically derived using MATLAB, with the parameters $\mathcal{s}, \mathcal{t}$ constrained to the range 1 – 5 and the resulting solutions are presented below.

Equation 1:

s	t	r	v
1	2	83	84
1	4	89	90
2	1	83	84
2	3	87	88
2	5	95	96
3	2	87	88
3	4	93	94
4	1	89	90
4	3	93	94
4	5	101	102
5	2	95	96
5	4	101	102

Fig.13

Equation 2:

s	t	r	v
1	2	371	372
1	4	377	378
2	1	371	372
2	3	375	376
2	5	383	384
3	2	375	376
3	4	381	382
4	1	377	378
4	3	381	382
4	5	389	390
5	2	383	384
5	4	389	390

Fig.14

Equation 3:

s	t	r	v
1	2	16571	16572
1	4	16577	16578
2	1	16571	16572
2	3	16575	16576
2	5	16583	16584
3	2	16575	16576
3	4	16581	16582
4	1	16577	16578
4	3	16581	16582
4	5	16589	16590
5	2	16583	16584
5	4	16589	16590

Fig.15

Equation 4:

s	t	r	v
1	2	20303	20304
1	4	20309	20310
2	1	20303	20304
2	3	20307	20308
2	5	20315	20316
3	2	20307	20308
3	4	20313	20314
4	1	20309	20310
4	3	20313	20314
4	5	20321	20322
5	2	20315	20316
5	4	20321	20322

Fig.16

Equation 5:

s	t	r	v
1	2	20591	20592
1	4	20597	20598
2	1	20591	20592
2	3	20595	20596
2	5	20603	20604
3	2	20595	20596
3	4	20601	20602
4	1	20597	20598
4	3	20601	20602
4	5	20609	20610
5	2	20603	20604
5	4	20609	20610

Fig.17

Equation 6:

s	t	r	v
1	2	36791	36792
1	4	36797	36798
2	1	36791	36792
2	3	36795	36796
2	5	36803	36804
3	2	36795	36796
3	4	36801	36802
4	1	36797	36798
4	3	36801	36802
4	5	36809	36810
5	2	36803	36804
5	4	36809	36810

Fig.18

3. CONCLUSIONS

The generalized quaternary quadratic Diophantine equation $s^2 + t^2 + r^2 = v^2 - (\mathcal{K} + \mathcal{P})$ with $\mathcal{K}$ being Krishnamurthy number and $\mathcal{P}$ being Leyland prime up to five digits, has been systematically investigated. Three distinct patterns were identified, each generating explicit families of integer solutions. These solutions were computed using MATLAB, providing both theoretical validation and computational confirmation.

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